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LARGE DEFLECTION RESPONSE OF ANNULAR PLATES ON PASTERNAK FOUNDATIONS

J. S. SMAILL
Department of Mechanical Engineering, University of Canterbury, Private bag, Christchurch, New Zealand

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Abstract—The influence of the elastic foundation configuration on the large deflection axisymmetric response of cylindrically orthotropic thin annular plates is examined for uniformly distributed loads. The solution of the dynamic form of the Von Kármán type equations governing the behaviour of the system is obtained using a fourth order finite difference representation for the spatial domain with the Newmark-β scheme being used for the time domain. Results for the fixed edge and simply supported immovable edge boundary conditions for a plate on a Pasternak foundation with or without an annular cut-out are presented for both the static and step loading cases. The inner edge boundary conditions for these configurations of a Pasternak foundation are examined and the apparent anomaly of the annular foundation increasing the stiffness in the recent literature addressed. The significance of the foundation parameters on plate response as well as the geometric non-linearity is considered.

1. INTRODUCTION

With the economic demand for material efficiency combined with the need to improve the resistance of structures to large dynamic loads such as can occur due to earthquakes, industrial explosion or wave action in offshore structures there has been an increasing interest in large deflection dynamic analysis. Recent earthquake experience (Pender and Robertson, 1987; Rutledge, 1988) has focussed attention on the mounting and anchoring of structures such as storage tanks and pressure vessels as well as heavy duty equipment. This often involves the use of annular plates in conjunction with elastic foundations.

Sinha (1963), using the Berger assumption (1955) to effectively decouple the governing equations describing the plate behaviour, investigated the geometric non-linear static deflection of an isotropic circular plate on a Winkler foundation based on a series solution. A single term Galerkin technique was applied by Datta (1974) and Banerjee (1976) with the value of the Berger constant being determined by the specific boundary conditions following appropriate transformation.

The dynamic geometric non-linear response of an isotropic circular plate on a Winkler–Pasternak foundation has been studied by Nath (1982) by extending the technique of Alwar and Nath (1977) to include the foundation reaction. A backward difference Taylor series expansion was used to linearize the dynamic form of the von Kármán equations for the plate/foundation combination with the spatial domain being described by a series solution using Chebyshev polynomials, the associated recurrence relationships and the specified boundary conditions. The time domain was represented by the Houbolt four-point algorithm with the resultant set of equations solved to determine the coefficients for the Chebyshev polynomials.

An alternative solution for an orthotropic plate on a Winkler–Pasternak foundation was presented by Dumir (1987) using an orthogonal point collocation representation based upon the zeros of a Legendre polynomial for the spatial domain with the Newmark-β scheme defining the time domain.

The solution for orthotropic annular plates on elastic foundation was presented by Nath and Jain (1983) using the Chebyshev polynomials and the implicit Houbolt scheme with the inner boundary condition being that associated with an annular foundation. Unfortunately the majority of the data presented are of a vertical deflection amplitude that results in the in-plane deflection contribution being negligible. Recently, Dumir (1988) extended the orthogonal point collocation method to consider orthotropic annular plates on an elastic foundation. In addition an energy method to determine the maximum transient deflection, using results from a static analysis, was presented. The selection of an inner boundary condition appropriate to a continuous foundation, while stating that the study presented results for an annular foundation, gave a deflection for the Pasternak foundation which was less than that reported by Nath for equivalent load cases. Dumir attributed these differences to Nath having used a continuous foundation.

To clarify this apparent anomaly the present investigation was carried out using finite differences to represent the spatial domain and a Newmark-β recurrence scheme based upon the finite element representation of the time domain, after Zienkiewicz (1977). The dynamic von Kármán equations were solved for the Pasternak foundation with the geometric non-linearity being implemented as a pseudo-load. Iteration to convergence was carried out at each time step. Both the continuous and annular foundation were considered and the resultant data examined in relation to previously reported information in the literature.

2. GOVERNING EQUATION

The model used to define the reaction at the plate/foundation interface was

$$p(r,t) = k_1 w + k_3 w^3 - g\nabla^2 w$$

incorporating the well known cases used by Dumir (1988)

$k_1 \neq 0,\ k_3, g = 0$	The Winkler foundation
$k_1 \geqslant 0,\ g \neq 0,\ k_3 = 0$	The Pasternak foundation (1954)
$k_1,\ k_3 \geqslant 0,\ g = 0$	The Nonlinear Winkler Foundation following Massalas and Kafousias (1979).

While the inertia of the foundation could be included, following Vlasov and Leontier (1966), by the use of an effective mass density for the plate, to simplify the case for the continuous foundation the assumption that the foundation was massless was adopted. It should be noted that the effective incorporation of the foundation mass modifies the relative dimensionless time parameter but does not influence the resultant amplitude of the deflection.

The two foundation cases considered were a continuous foundation (Fig. 1a) and an annular foundation (Fig. 1b). The inner boundary for the plate was a free edge condition

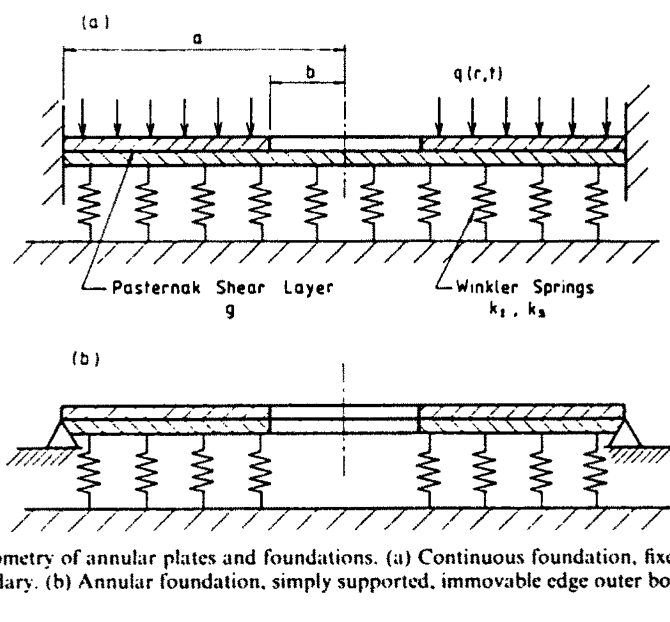

Fig. 1. Geometry of annular plates and foundations. (a) Continuous foundation, fixed edge outer boundary. (b) Annular foundation, simply supported, immovable edge outer boundary.

while the outer boundary conditions considered were the fixed edge boundary and simply supported immovable edge conditions.

The governing equation for the plate/foundation combination after Huang (1972) was

$$D\left[w_{,rrrr}+\frac{2}{r}w_{,rrr}-\frac{\beta}{r^2}w_{,rr}+\frac{\beta}{r^3}w_{,r}\right]-q(r,t)+mhw_{,tt}+mhk_v w_{,t}+p(r,t)$$

$$-\frac{E_\theta h}{\beta-\nu_\theta^2}\left[\left(w_{,rr}+\frac{1}{r}w_{,r}\right)\left(u_{,r}+\frac{\nu_\theta}{r}u+\tfrac{1}{2}(w_{,r})^2\right)+(w_{,r})\left(u_{,rr}+\frac{\nu_\theta}{r}u_{,r}-\frac{\nu_\theta}{r^2}u+(w_{,r})(w_{,rr})\right)\right]=0$$

$$u_{,rr}+\frac{1}{r}u_{,r}-\beta\frac{u}{r^2}+(w_{,r})(w_{,rr})+\frac{1-\nu_\theta}{2r}(w_{,r})^2=0$$

where a = outer radius; b = inner radius; h = plate thickness; k_v = viscous damping; m = plate density; $p(r,t)$ = load; $q(r,t)$ = foundation reaction; r = radial position; u = in-plane deflection; w = vertical deflection; D = flexural rigidity; β = orthotropic parameter;

$$D=\frac{E_\theta h^3}{12(\beta-\nu_\theta^2)};\qquad \beta=\frac{E_\theta}{E_r}=\frac{\nu_\theta}{\nu_r}.$$

with E_θ, E_r and ν_θ, ν_r being the elastic moduli and Poisson's ratio for the plate material in the circumferential and radial directions respectively. A subscript variable following a comma denotes partial differentiation with respect to the variable.

Substituting for the foundation reaction the non-dimensional form of the governing equations becomes

$$\alpha_{,\rho\rho\rho\rho}+\frac{2}{\rho}\alpha_{,\rho\rho\rho}-\frac{\beta}{\rho^2}\alpha_{,\rho\rho}+\frac{\beta}{\rho^3}\alpha_{,\rho}+\alpha_{,\tau\tau}+\chi_v\alpha_{,\tau}-\frac{12(\beta-\nu_\theta^2)}{\beta}\left[\varepsilon+K_1\alpha+K_3\alpha^3-G\left(\alpha_{,\rho\rho}+\frac{1}{\rho}\alpha_{,\rho}\right)\right]$$

$$-12\delta\left[\left(\alpha_{,\rho\rho}+\frac{1}{\rho}\alpha_{,\rho}\right)\left(\zeta_{,\rho}+\frac{\nu_\theta}{\rho}\zeta+\frac{1}{2\delta}(\alpha_{,\rho})^2\right)+(\alpha_{,\rho})\left(\zeta_{,\rho\rho}+\frac{\nu_\theta}{\rho}\zeta_{,\rho}-\frac{\nu_\theta}{\rho^2}+\frac{1}{\delta}(\alpha_{,\rho})(\alpha_{,\rho\rho})\right)\right]=0$$

$$\zeta_{,\rho\rho}+\frac{1}{\rho}\zeta_{,\rho}-\beta\frac{\zeta}{\rho^2}+\frac{1}{\delta}(\alpha_{,\rho})(\alpha_{,\rho\rho})+\frac{1-\nu_\theta}{2\delta\rho}(\alpha_{,\rho})=0.0$$

where $\alpha=w/h$; $\rho=r/a$; $\delta=a/h$; $\zeta=u/h$; $\tau=t(D/mha^4)^{1/2}$; $\chi=k_v(mha^4/D)^{1/2}$; $G=g(a^2/E_r)$; $K_1=k_1(a^4/E_r)$; $\xi=b/a$; $K_3=k_3(a^4/E_rh^2)$; $\varepsilon(\rho,\tau)=q(r,t)(a^4/Eh^4)$.

2.1. *Boundary conditions*

The boundary conditions used to solve the governing equations were as follows.

2.1.1. *Inner boundary* $\rho=\xi$. Since the inner edge is a free boundary the radial in-plane stress is equated to zero, i.e.

$$\zeta_{,\rho}+\frac{\beta\nu_\theta}{\rho}+\frac{1}{2\delta}(\alpha_{,\rho})^2=0.0.$$

For the linear and non-linear Winkler foundation models the resultant deflection was independent of the foundation configuration. For the Pasternak model the response was a function of whether the associated "shear layer" was continuous at the inner boundary or discontinuous, because this determined the generalized shear force acting. The model proposed by Dumir (1988) for the annular foundation does not allow for discontinuity in the derivative ($\alpha_{,\rho}$) with respect to the foundation and therefore the resultant values of deflection given are appropriate to the continuous foundation *not* the annular foundation.

(i) Continuous foundation
The generalized shear force for the inner free edge is given by

$$\phi = \left(\alpha_{,\rho\rho\rho} + \frac{1}{\rho}\alpha_{,\rho\rho} - \frac{\beta}{\rho^2}\alpha_{,\rho} - \frac{12(\beta - v_\theta)}{\beta} G\alpha_{,\rho}\right) = 0.$$

(ii) Annular foundation

$$\phi = \left(\alpha_{,\rho\rho\rho} + \frac{1}{\rho}\alpha_{,\rho\rho} - \frac{\beta}{\rho^2}\alpha_{,\rho}\right) = 0.$$

2.1.2. *Outer boundary* $\rho = 1$.

(i) Clamped immovable edge condition

$$\alpha = 0, \quad \frac{\partial\alpha}{\partial\rho} = 0, \quad \frac{\partial^3\alpha}{\partial\rho^3} = 0, \quad \zeta = 0.$$

(ii) Simply supported immovable edge condition

$$\alpha = 0, \quad \frac{\partial^2\alpha}{\partial\rho^2} + \frac{v}{\rho}\frac{\partial\alpha}{\partial\rho} = 0, \quad \zeta = 0.$$

3. NUMERICAL SOLUTION

A finite difference representation for the spatial domain was used to numerically solve the governing equations for the given boundary conditions while a recursive scheme was adopted to model the time domain following Zienkiewicz (1977).

The geometric non-linear contribution was implemented as a "pseudo-load" associated with the forcing function. Therefore the only matrix inversions required were those associated with establishing the initial finite difference schemes for the transverse and in-plane deflection of the plate. The initial solution for transverse deflection was calculated using the in-plane deflection from the preceding time step and iteration based upon the updated "pseudo-load" carried out until the convergence criterion was satisfied. The convergence criterion adopted was that the variation for the vertical deflection between successive iterations was less than 0.1%.

The governing equation in the spatial domain for the transverse deflection at any time τ was

$$\mathbf{M}\boldsymbol{\alpha}_{,\tau\tau} + \mathbf{C}\boldsymbol{\alpha}_{,\tau} + \mathbf{K}\boldsymbol{\alpha} + \mathbf{F} = 0$$

where $\mathbf{M}$ = mass matrix; = identity matrix because of non-dimensional time parameter used; $\mathbf{C}$ = viscous damping coefficient; $\mathbf{K}$ = matrix of coefficients; $\boldsymbol{\alpha}$ = transverse displacement vector; $\mathbf{F}$ = the forcing function containing "pseudo-load".

The time domain was represented by the following generalized three-point recursive relationship

$$\begin{aligned}[\mathbf{M} + \gamma\Delta\tau\mathbf{C} + \beta\Delta\tau^2\mathbf{K}]\boldsymbol{\alpha}_{n+1} + [-2\mathbf{M} + (1-2\gamma)\Delta\tau\mathbf{C} + (\tfrac{1}{2}+\gamma-2\beta)\Delta\tau^2\mathbf{K}]\boldsymbol{\alpha}_n \\ + [\mathbf{M} + (\gamma-1)\Delta\tau\mathbf{C} + (\tfrac{1}{2}-\gamma+\beta)\Delta\tau^2\mathbf{K}]\boldsymbol{\alpha}_{n-1} + (\beta\Delta\tau^2)\mathbf{F}_{n+1} \\ + (\tfrac{1}{2}+\gamma-2\beta)\Delta\tau^2\mathbf{F}_n + (\tfrac{1}{2}-\gamma+\beta)\Delta\tau^2\mathbf{F}_{n-1} = 0\end{aligned}$$

and numerical experimentation was carried out to determine the optimum procedure for the specific solution scheme. It was established that the most efficient solution scheme was

$\gamma = \frac{1}{2}$, $\beta = \frac{1}{4}$ while the time step was taken as $\Delta\tau = 0.002$. The solutions were obtained using $\nu_\theta = 0.25$.

The static deflections for the clamped outer boundary condition were obtained by solution of the governing equation for the spatial domain. Because of numerical instability associated with the "pseudo-load" technique at large deflections, in the case of the simply supported outer boundary, the static solution was obtained by solving the appropriate step loading case with a viscous damping coefficient $\chi = 20$ after Nath (1982). The resultant deflection, following damping of the oscillatory motion, was taken as the static result.

4. RESULTS AND CONCLUSIONS

Transient results from this study are compared with those presented by Nath and Jain (1983) and Dumir (1988) in Table 1. It should be noted that the results of Nath *et al.* are associated with the annular foundation inner edge boundary condition for the Pasternak foundation *not* the continuous foundation as suggested by Dumir. The inner boundary condition applied by Dumir for "zero generalized shear force V_r" is applicable to the continuous Pasternak foundation, explaining the unexpected increase in stiffness reported for the plate/foundation system in this later paper.

Making appropriate transformations of these reported results in Table 1 it can be seen that good agreement was achieved with the existing literature. It is worthy of note that the deflection amplitudes used in this comparison mean that the in-plane contribution is negligible, as can be seen from the results included from linear numerical analysis.

A better comparison of solution adequacy, involving in-plane deflection under both boundary conditions, is shown in Tables 2 and 3 for the continuous and annular foundations. Examination of the corresponding linear results for these foundation cases establishes the significance of in-plane deflection. Again, good agreement is achieved between the results of Dumir (1988) attributed to the "annular" foundation with those for the continuous foundation.

Since the linear and non-linear Winkler model implies no interaction between adjacent sections of the foundation, the results reported by Nath and Jain (1983) and Dumir (1988) are independent of the foundation configuration for this characteristic. However, the Pasternak model, because of the "shear layer", exhibits interaction between adjacent foundation sections; therefore the resultant deflection is a function of whether the foun-

Table 1. Comparison of maximum transient response $\alpha(\xi)_{max}$ ($\varepsilon = 5$, $\xi = 1/3$, $\beta = 0.50$, $\nu_\theta = 1/3$)

		Annular foundation				Continuous foundation		
				Self			Self	
k'	g'	Nath and Jain	Dumir†	Non-linear	Linear	Dumir†	Non-linear	Linear
Fixed edge outer boundary condition								
100	0	0.2630	0.2624	0.2639	0.2644	0.2624	0.2639	0.2644
100	25	0.1715	0.1711	0.1713	0.1714	0.1164	0.1177	0.1177
100	50	0.1288	0.1288	0.1281	0.1282	0.0771	0.0780	0.0780
150	0	0.1934	0.1937	0.1927	0.1928	0.1937	0.1927	0.1928
150	50	0.1055	0.1056	0.1054	0.1054	0.0691	0.0685	0.0685
Simply supported outer boundary condition								
100	0	0.3104	0.3112	0.3108	0.3114	0.3112	0.3108	0.3114
100	25	0.1933	0.1930	0.1962	0.1963	0.1416	0.1420	0.1420
100	50	0.1427	0.1428	0.1473	0.1474	0.0924	0.0928	0.0928
150	0	0.2029	0.2024	0.2018	0.2020	0.2029	0.2024	0.2020
150	50	0.1137	0.1138	0.1167	0.1167	0.0793	0.0795	0.0795

$k' = \dfrac{k[(a-b)^4]}{D}$, $g' = \dfrac{g[(a-b)^2]}{D}$.

† The columns have been transposed with respect to the presentation in Dumir (1988).

Table 2. Maximum deflection for fixed edge plate $\alpha(\xi)_{max}$ ($\varepsilon = 15$, $\xi = 1/3$, $\nu_\theta = 1/3$)

K	G	Continuous foundation			Annular foundation	
		Dumir†	Non-linear	Linear	Non-linear	Linear
Isotropic case, $\beta = 1$						
0	0	1.999	1.994	4.001	1.994	4.001
5	0	1.769	1.758	2.928	1.758	2.928
10	0	1.566	1.566	2.262	1.566	2.262
5	1	1.344	1.349	1.663	1.609	2.348
10	1	1.203	1.202	1.377	1.423	1.876
5	2	1.053	1.054	1.151	1.460	1.932
10	2	0.944	0.947	1.011	1.290	1.584
Orthotropic case, $\beta = 3$						
0	0	1.525	1.528	2.667	1.528	2.667
5	0	1.383	1.379	2.102	1.379	2.102
10	0	1.249	1.250	1.696	1.250	1.696
5	1	1.118	1.120	1.363	1.247	1.690
10	1	1.007	1.011	1.168	1.127	1.395
5	2	0.912	0.915	1.015	1.126	1.395
10	2	0.834	0.830	0.903	1.017	1.185

† See Table 1.

dation is continuous or annular. To simplify the comparison only results for the Pasternak foundation are considered.

The maximum deflection for the isotropic plate with a clamped or fixed edge outer boundary on an annular foundation is shown by the dashed lines in Fig. 2 as a function of uniformly distributed step loading, while the static deflection for the equivalent uniform load is represented by the solid lines. The results for the continuous foundation can be seen in Fig. 3. The close agreement with the approximate maximum deflection reported by Dumir (1988) (solid symbols) is apparent. The response for orthotropic annuli can be seen in Figs 4 and 5 for the respective foundation configurations.

For the simply supported outer boundary edge the isotropic annuli results are given in Figs 6 and 7 while those for the orthotropic annular plates are presented in Figs 8 and 9 for the annular and continuous foundations respectively. In all cases the annular dimension

Table 3. Maximum deflection for simply supported plate $\alpha(\xi)_{max}$ ($\varepsilon = 10$, $\xi = 1/3$, $\nu_\theta = 1/3$)

K	G	Continuous foundation			Annular foundation	
		Dumir†	Non-linear	Linear	Non-linear	Linear
Isotropic case, $\beta = 1$						
0	0	2.263	2.290	17.323	2.290	17.323
5	0	1.964	1.984	4.726	1.984	4.726
10	0	1.691	1.706	2.745	1.706	2.745
5	1	1.461	1.474	2.028	1.699	2.879
10	1	1.240	1.248	1.481	1.422	1.926
5	2	1.091	1.103	1.254	1.469	2.040
10	2	0.940	0.944	1.012	1.236	1.488
Orthotropic case, $\beta = 3$						
0	0	1.752	1.773	6.195	1.773	6.195
5	0	1.539	1.556	3.115	1.556	3.115
10	0	1.345	1.356	2.045	1.356	2.045
5	1	1.206	1.213	1.650	1.326	1.991
10	1	1.042	1.049	1.251	1.148	1.456
5	2	0.944	0.950	1.091	1.137	1.460
10	2	0.824	0.829	0.902	0.984	1.130

† See Table 1.

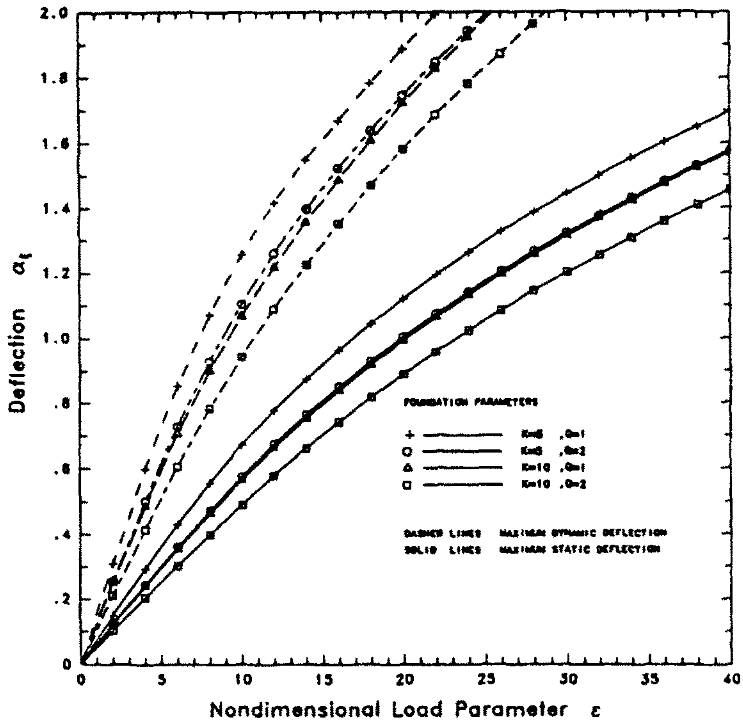

Fig. 2. Deflection response of isotropic clamped plate on annular Pasternak foundation.

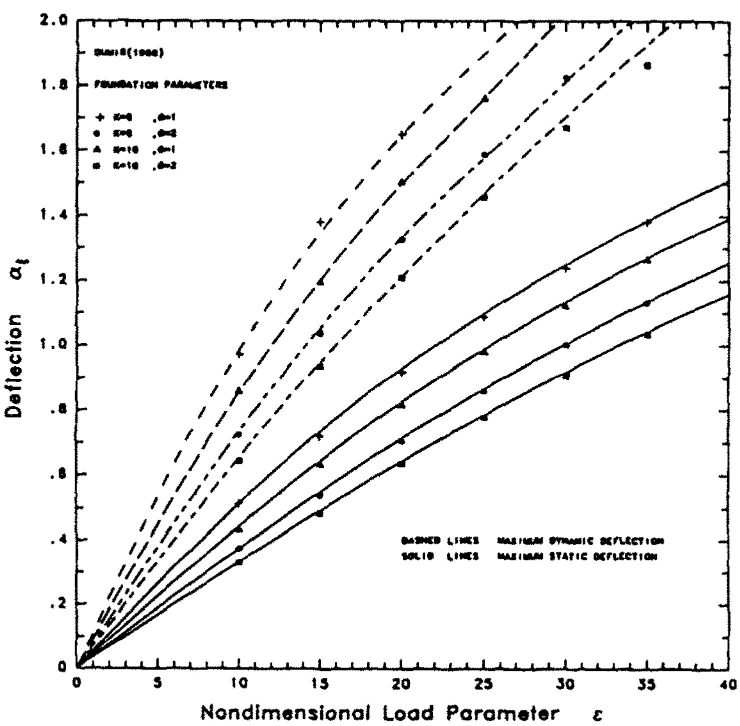

Fig. 3. Deflection response of isotropic clamped plate on continuous Pasternak foundation.

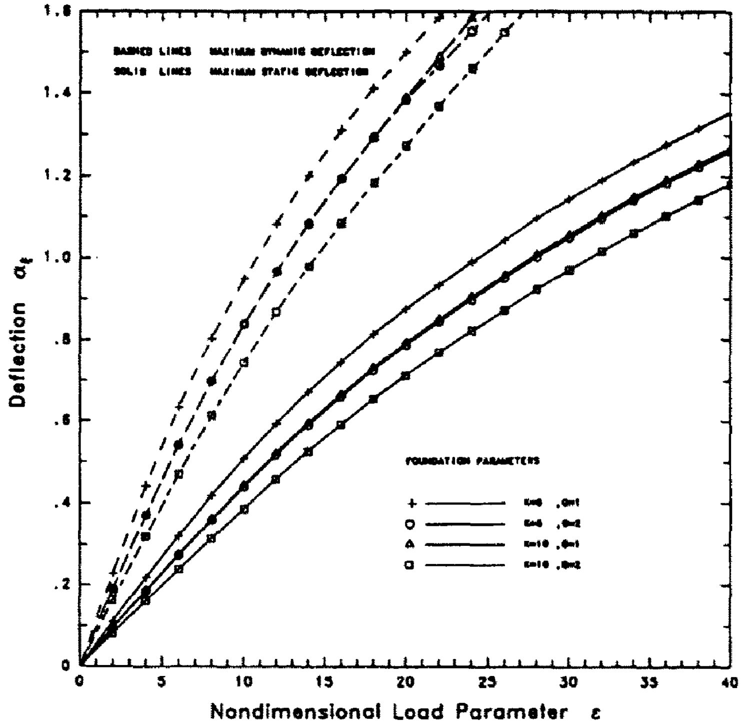

Fig. 4. Deflection response of orthotropic clamped plate on annular Pasternak foundation.

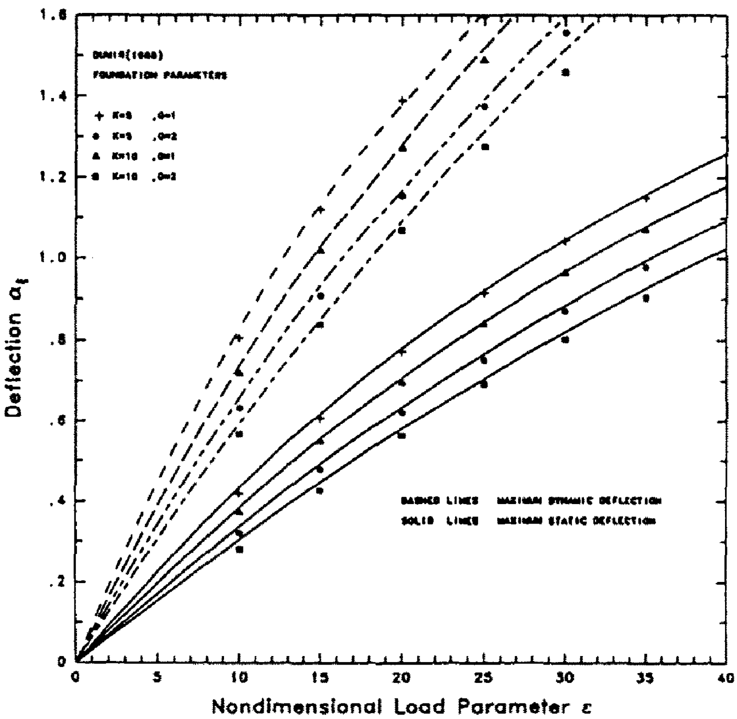

Fig. 5. Deflection response of orthotropic clamped plate on continuous Pasternak foundation.

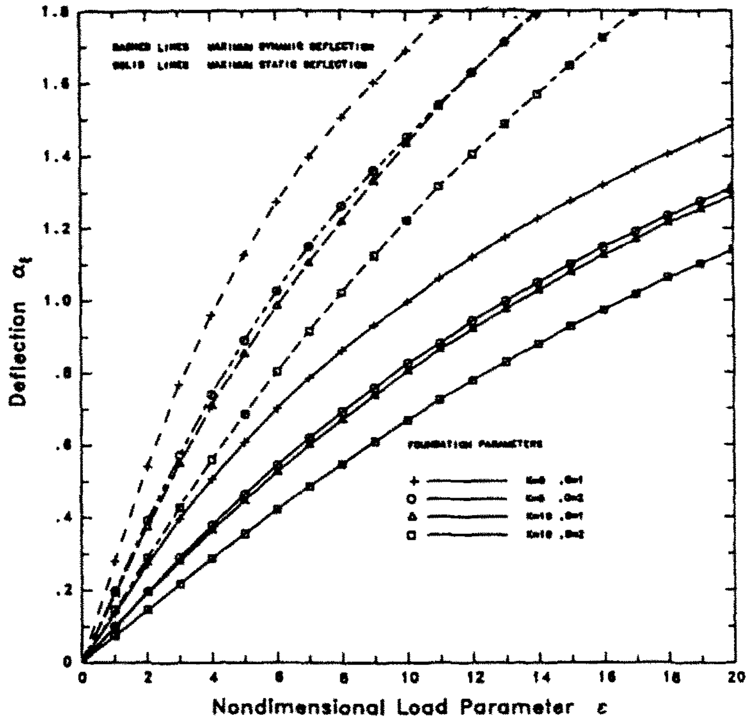

Fig. 6. Deflection response of isotropic simply supported plate on annular Pasternak foundation.

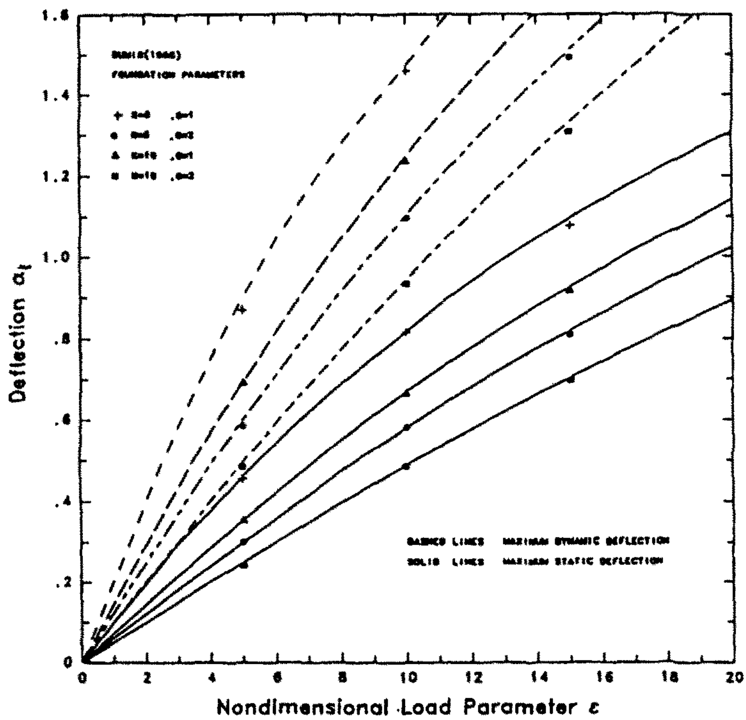

Fig. 7. Deflection response of isotropic simply supported plate on continuous Pasternak foundation.

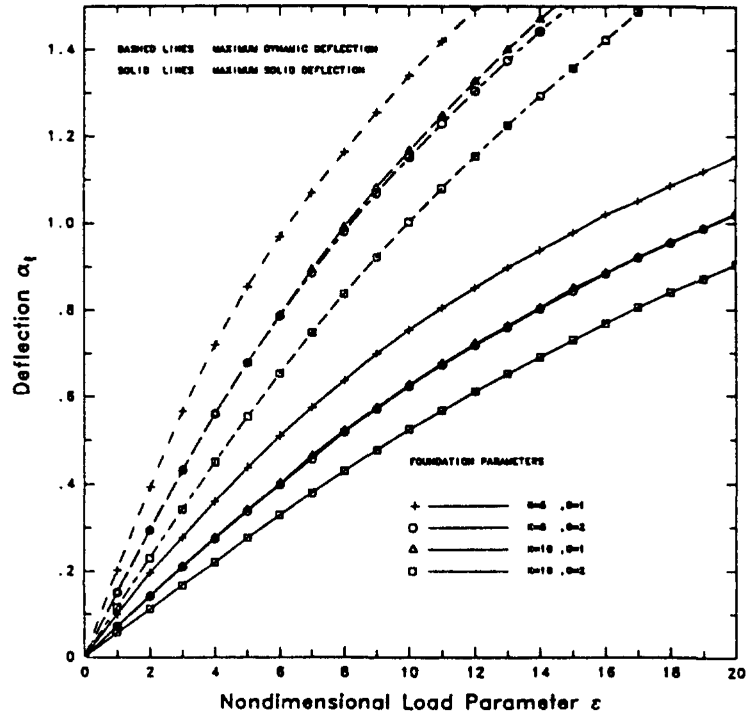

Fig. 8. Deflection response of orthotropic simply supported plate on annular Pasternak foundation.

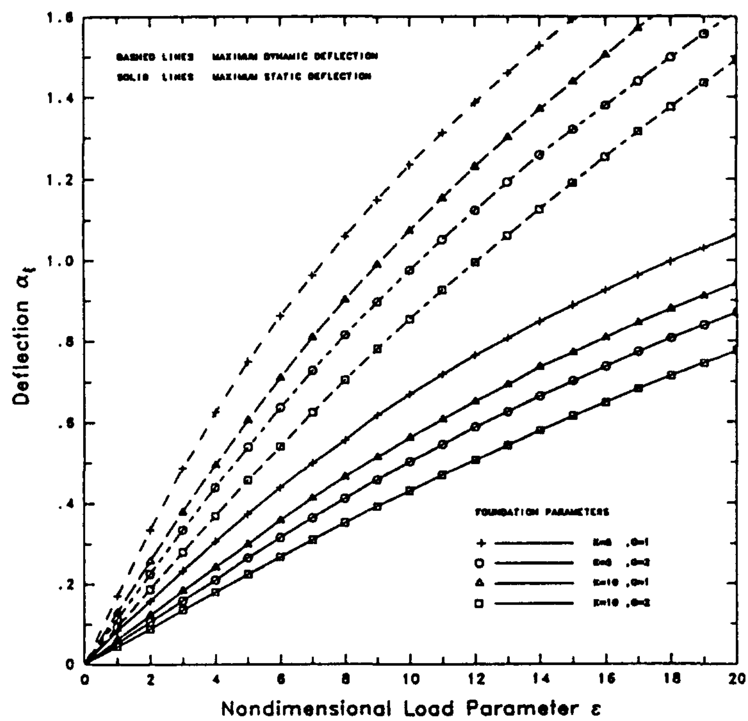

Fig. 9. Deflection response of orthotropic simply supported plate on continuous Pasternak foundation.

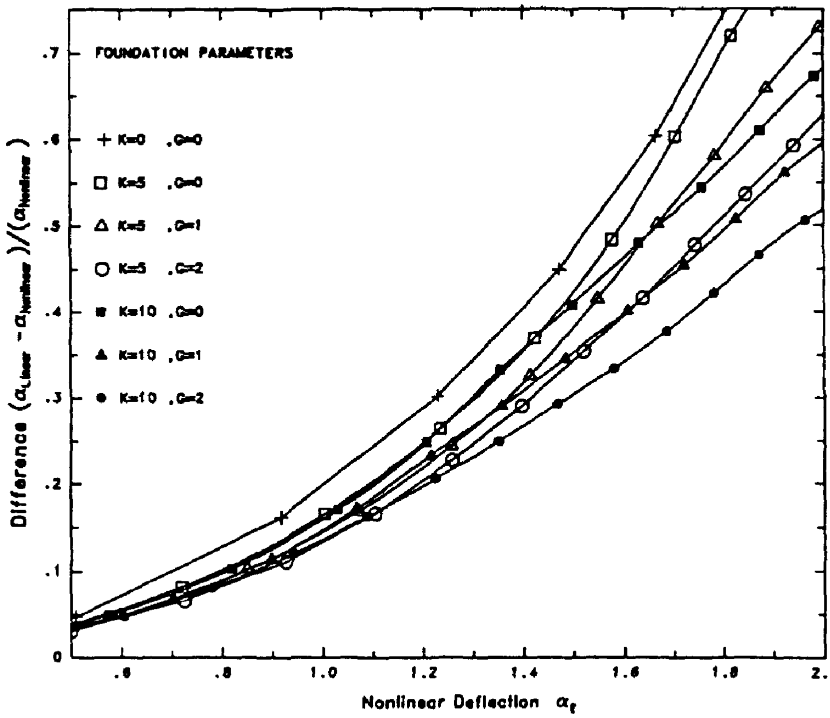

Fig. 10. Difference between linear and geometric non-linear deflection analysis for clamped plate on annular foundation.

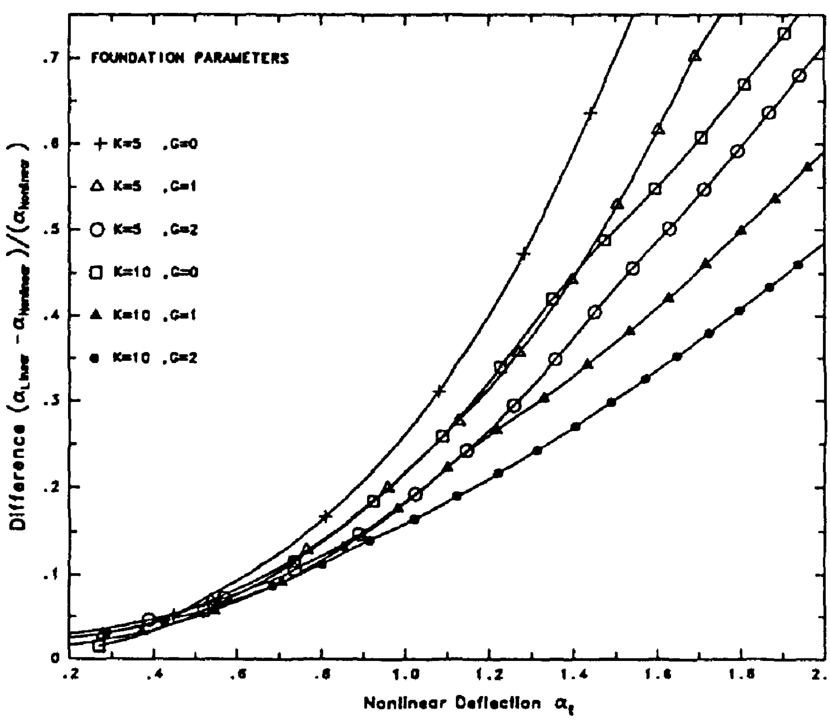

Fig. 11. Difference between linear and geometric non-linear deflection analysis for simply supported plate on annular foundation.

considered was $\xi = 1/3$ while the material properties where $v = 0.25$ for the isotropic plate and $\beta = 3$, $v_{\theta} = 0.25$ for the orthotropic case.

The decreasing deflection of the plate/foundation system as β, K and G increase confirms the observation of Dumir (1988). It is apparent from both the figures and Tables 2 and 3 that while the effect of the geometric non-linearity is to increase the plate stiffness, with increasing foundation stiffness this contribution to the rigidity becomes less significant. The shear parameter G in the Pasternak foundation can be seen to have more effect in decreasing deflection than the spring parameter K with the annular foundation case showing that an increase in K by 5 is approximately equivalent to increasing G by 1 for both outer boundary conditions as well as either an isotropic or orthotropic plate.

The greater significance of the foundation on the deflection of the simply supported plates can be seen in comparing Figs 2–5 with Figs 6–9 respectively, extending the observation of Dumir to the annular foundation.

The relative roles of the geometric non-linearity on the stiffness of the plate/foundation system as well as further confirmation of the significance of the foundation characteristic for the respective outer boundary conditions can be obtained by comparison of Figs 10 and 11. The fractional difference between the linear and non-linear numerical solution establishes the contribution of geometric non-linearity for the simply supported boundary condition with low foundation rigidity. The greater influence of higher foundation rigidity on deflection for these boundary conditions is also apparent.

It can be concluded that for both outer boundary conditions as well as isotropic and orthotropic plate materials the continuous foundation with a Pasternak characteristic has greater rigidity than the annular foundation.

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